M/EEG source analysis

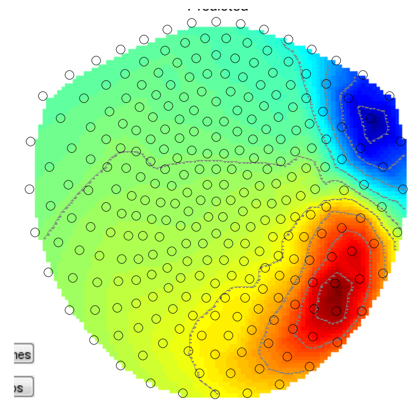
Gareth R. Barnes

Key points:

- What is an ill-posed inverse problem
- Prior knowledge- links to popular algorithms.
- Validation of prior knowledge/ Model evidence

The forward problem

Prediction



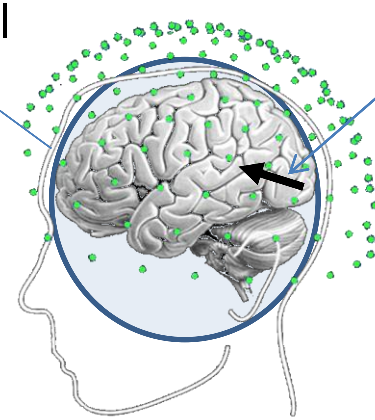
M/EEG sensors

Lead field (L) is the sensitivity of the M/EEG system to a dipolar source at a particular location

Lead fields (L)

Head Model

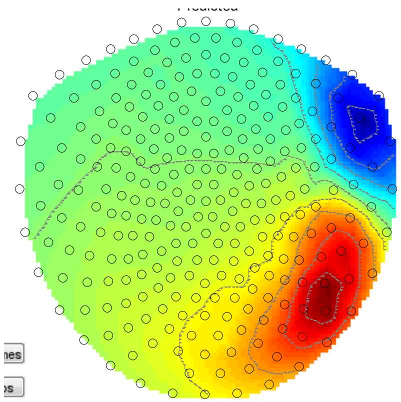
Dipolar source



Model describes conductivity & geometry

Analogy
2+3= ?

The Inverse problem



M/EEG
sensors

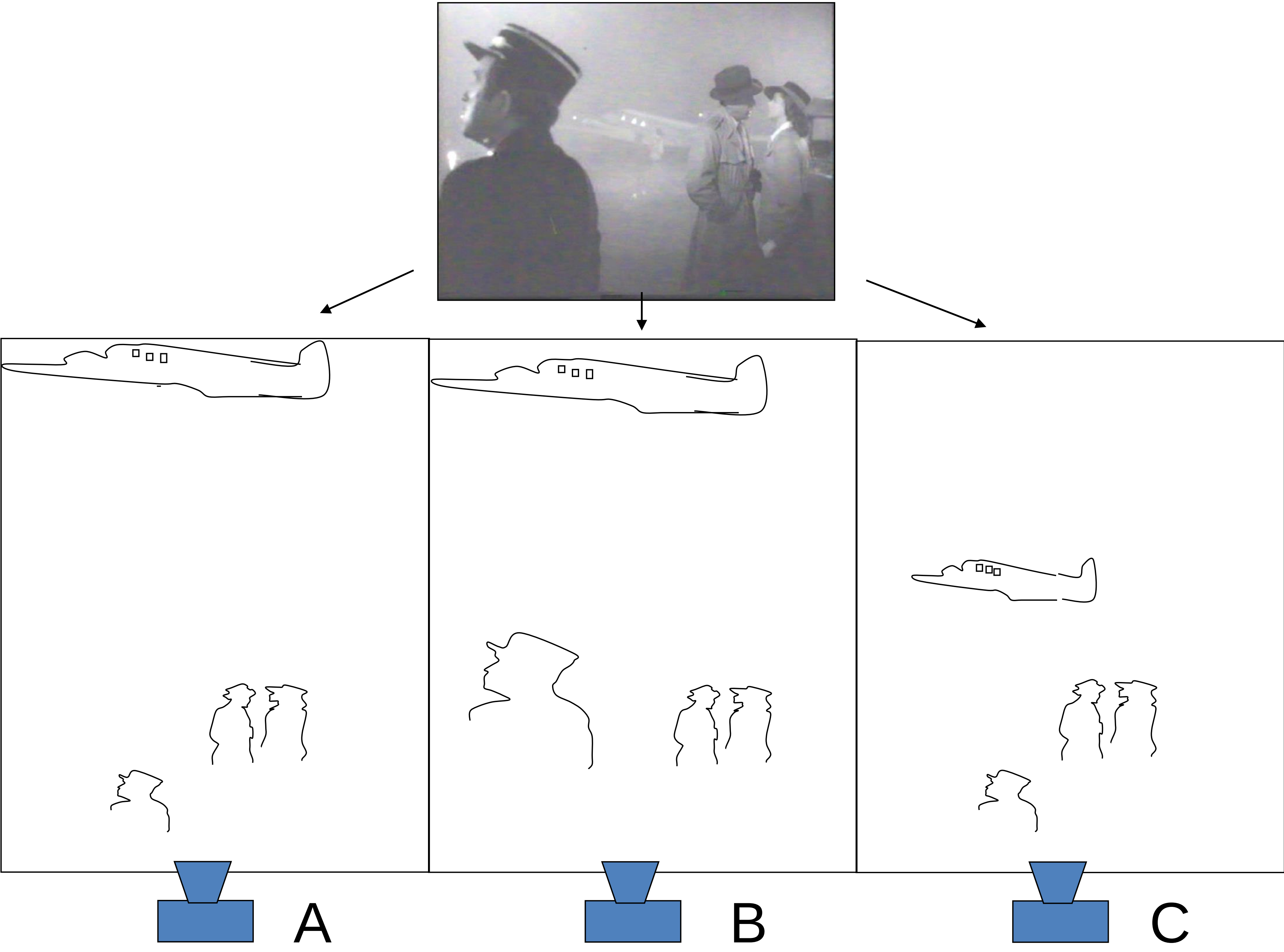
Measurement

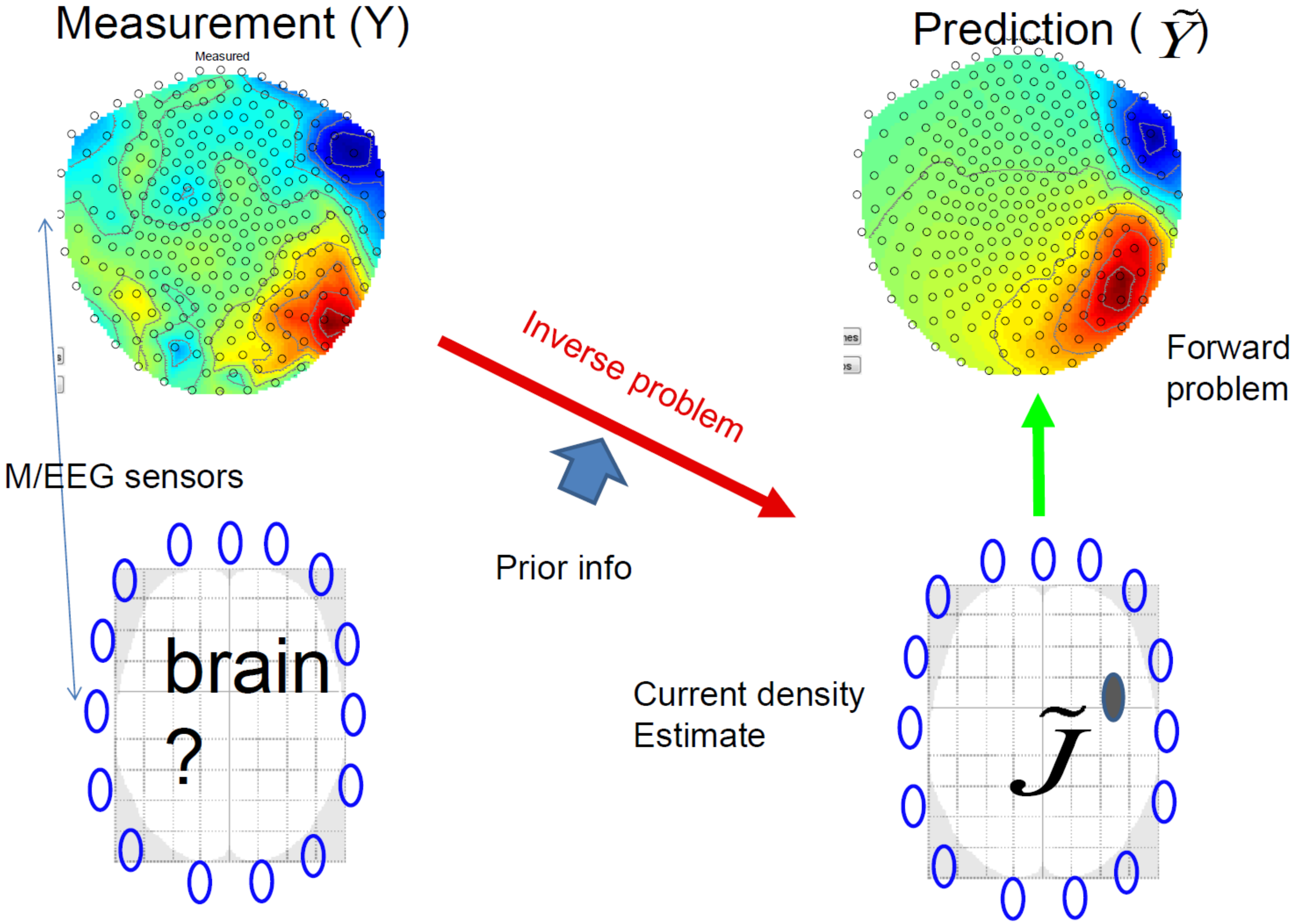
Which brain sources gave rise to these measured data ?

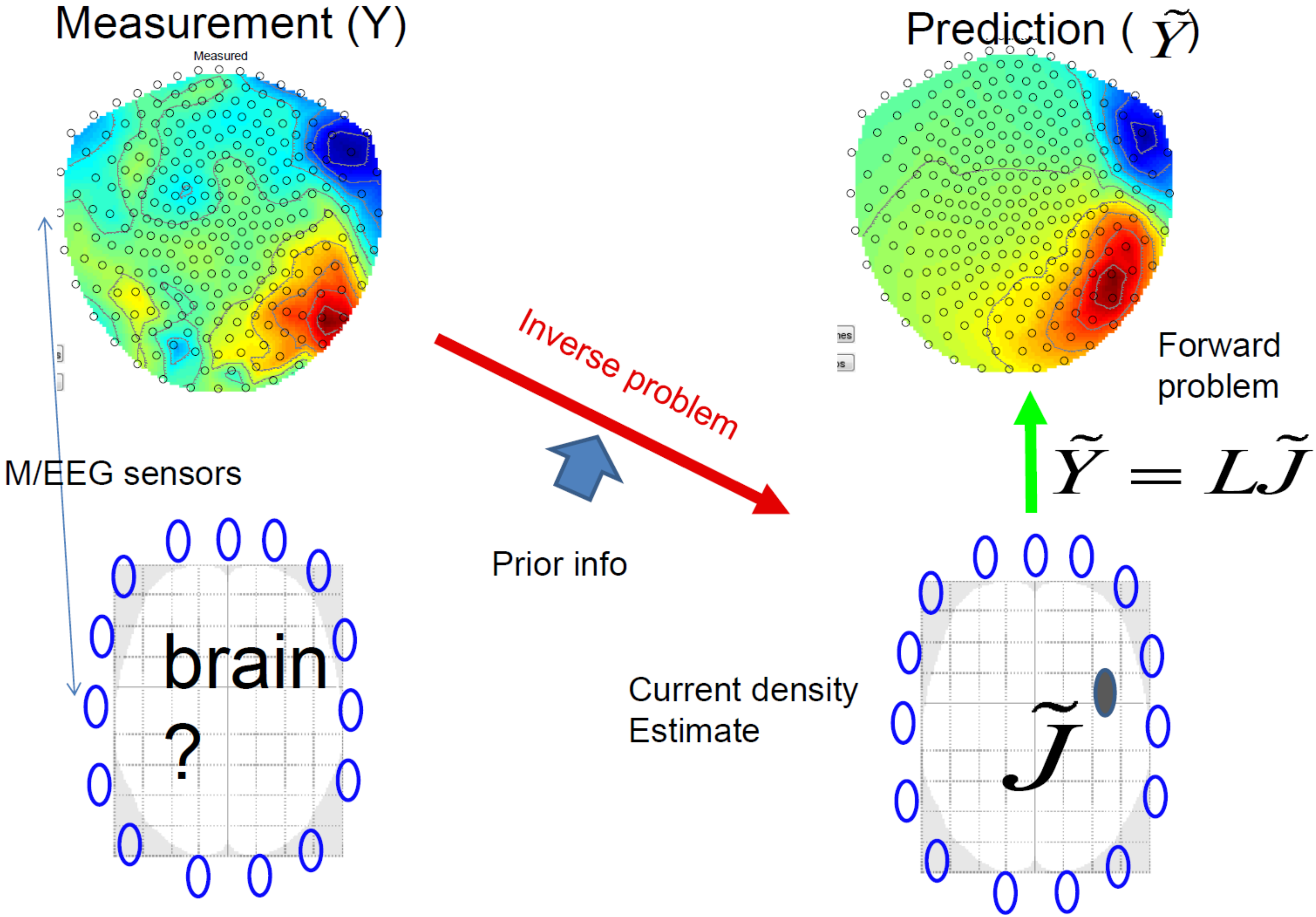
Inference

Analogy
5= ? +?

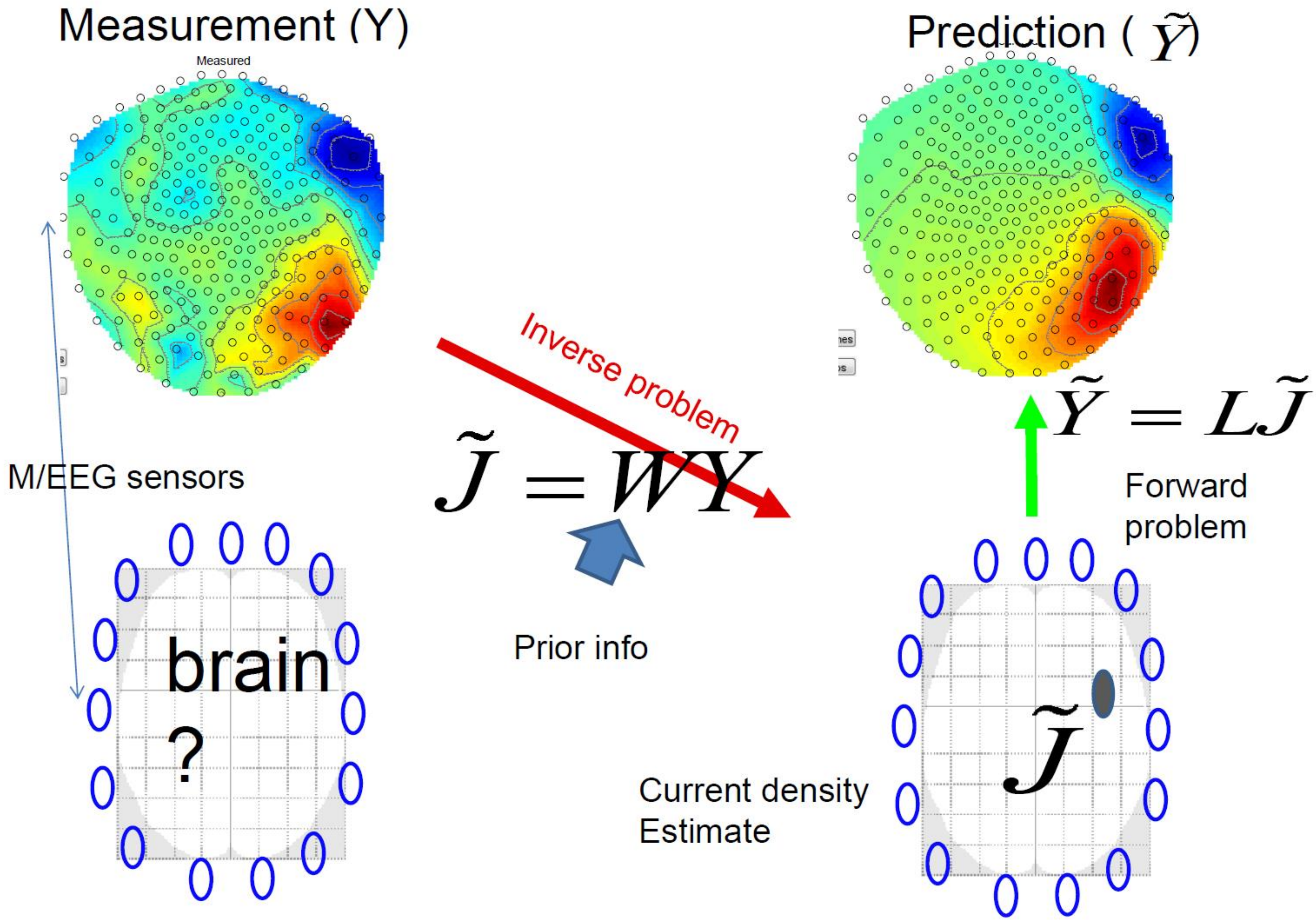

Inverse problems aren’t difficult


A
B
C


Measurement (Y)
Measured
Prediction ($\tilde{Y}$)
Inverse problem
Forward problem
M/EEG sensors
Prior info
brain ?
Current density Estimate
$\tilde{J}$



Measurement (Y)
Measured
Prediction ($\tilde{Y}$)
Inverse problem
Forward problem
$\tilde{Y} = L\tilde{J}$
M/EEG sensors
Prior info
brain ?
Current density Estimate
$\tilde{J}$

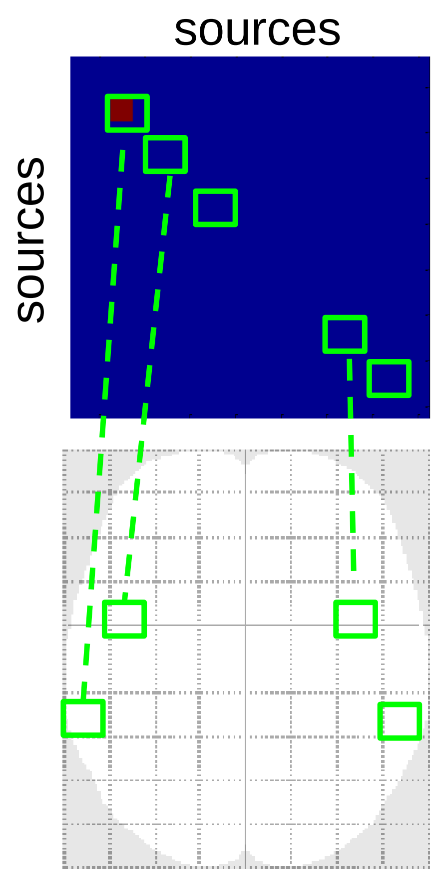

Measurement (Y)
Measured
Prediction ($\tilde{Y}$)
Inverse problem
$\tilde{J} = WY$
$\tilde{Y} = L\tilde{J}$
M/EEG sensors
Forward problem
Prior info
brain
?
Current density Estimate
$\tilde{J}$

Inversion depends on choice of source covariance matrix (prior information)

$$W = CL'(R + LCL')^{-1}$$

Sensor Noise (known)

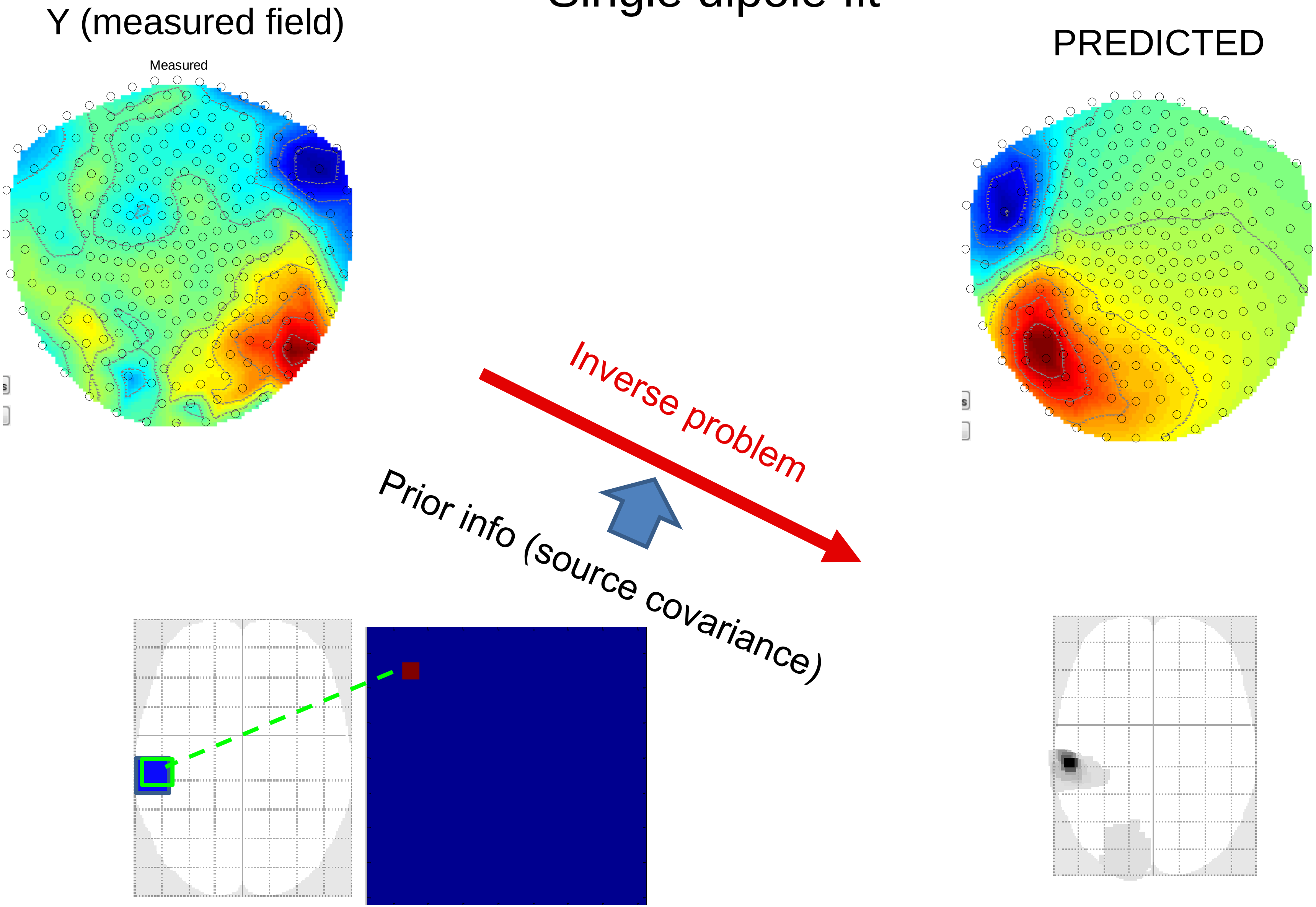
Lead field (known)



Source covariance matrix,
One diagonal element per source

Prior information

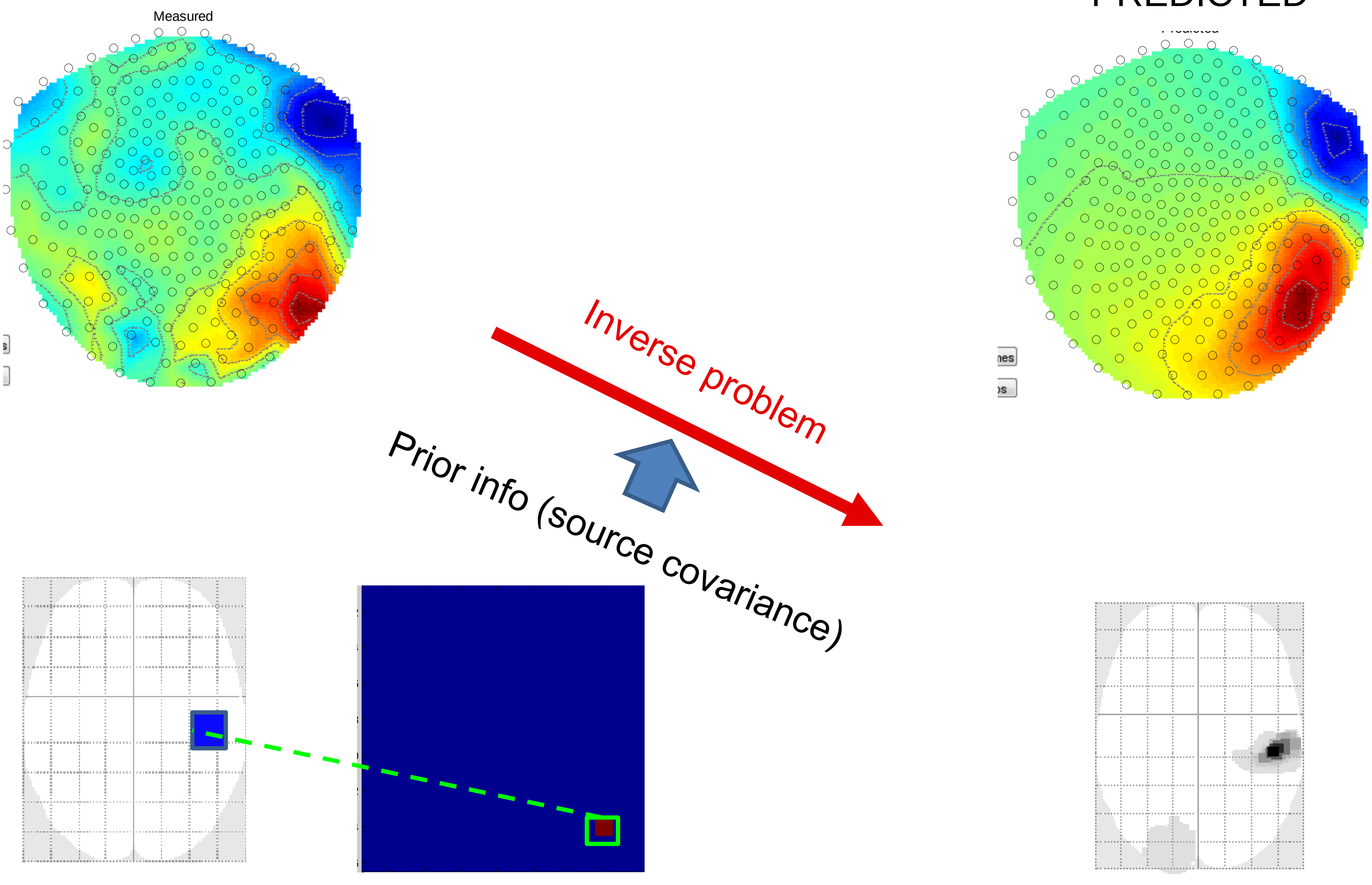
Single dipole fit



Single dipole fit

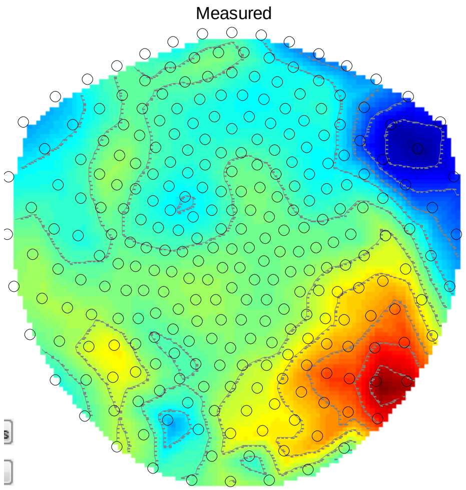
Y (measured field)

PREDICTED

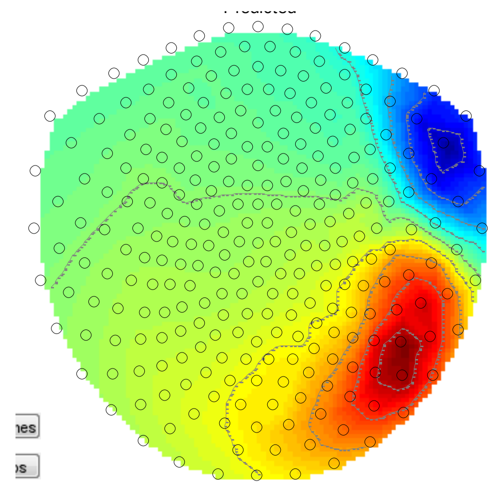


Two dipole fit

Y (measured field)

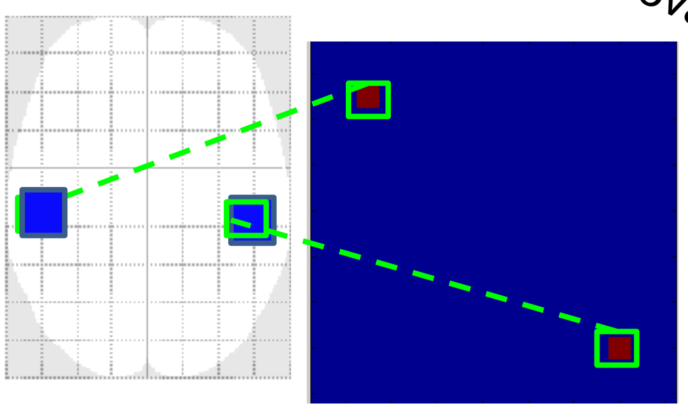


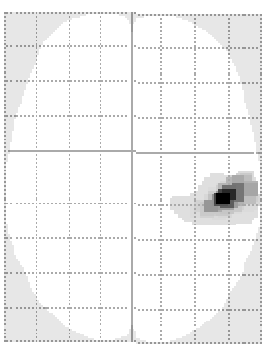
PREDICTED



Inverse problem

Prior info (source covariance)

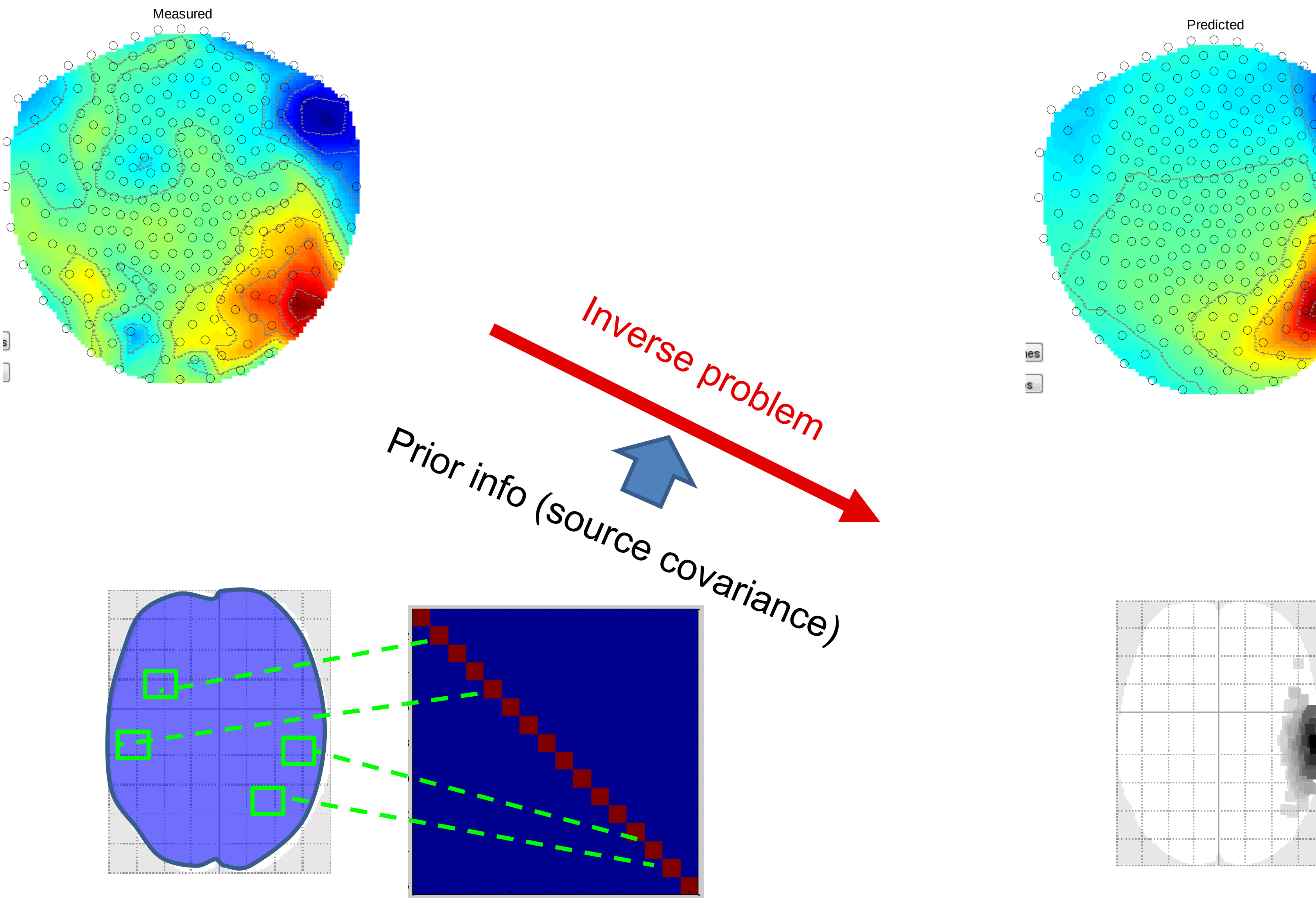




Minimum norm

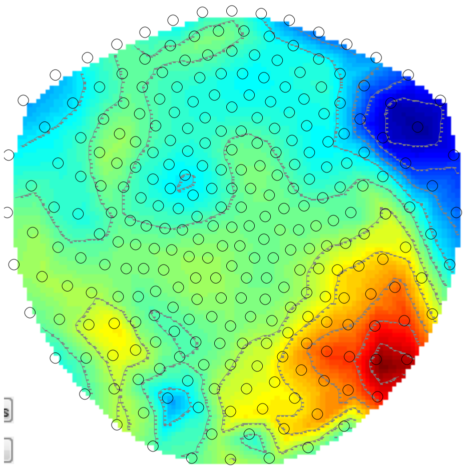
Y (measured field)

PREDICTED

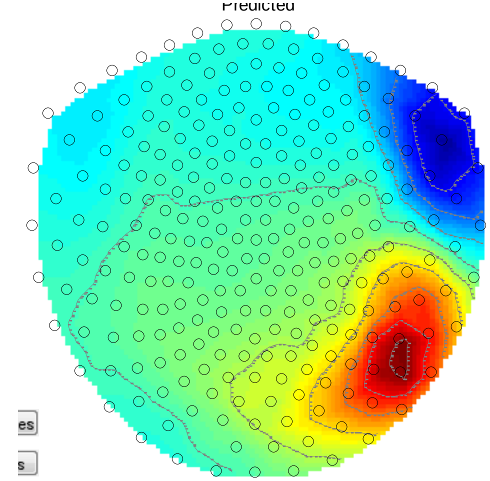


Beamformer (adaptive algorithm/ Empirical)

Y (measured field)



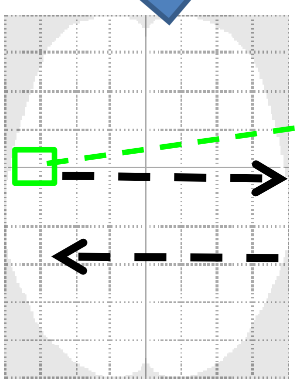
PREDICTED

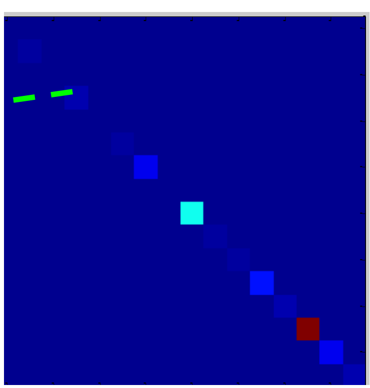


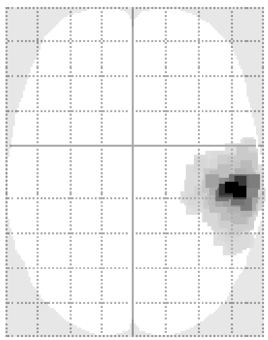
Inverse problem

Projection onto lead field*

Prior info (source covariance)



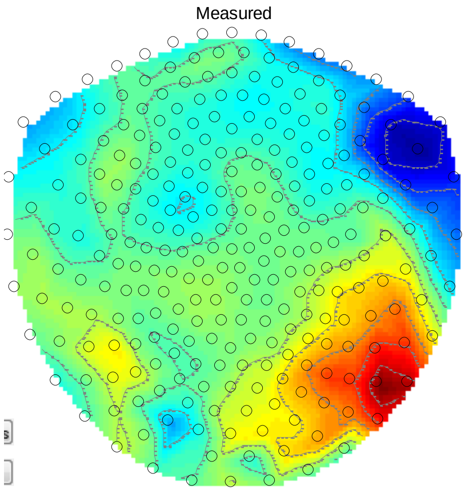




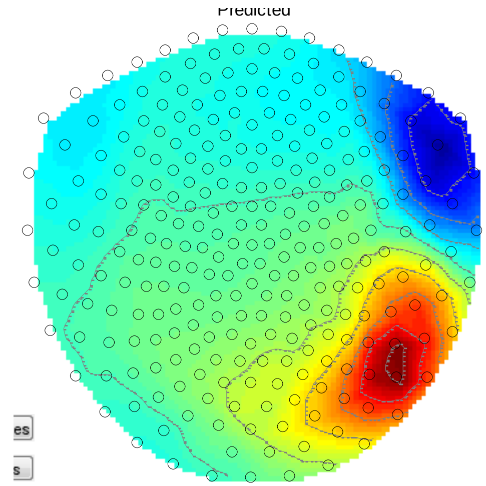
*Assuming no correlated sources

fMRI biased dSPM
(Dale et al. 2000)

Y (measured field)

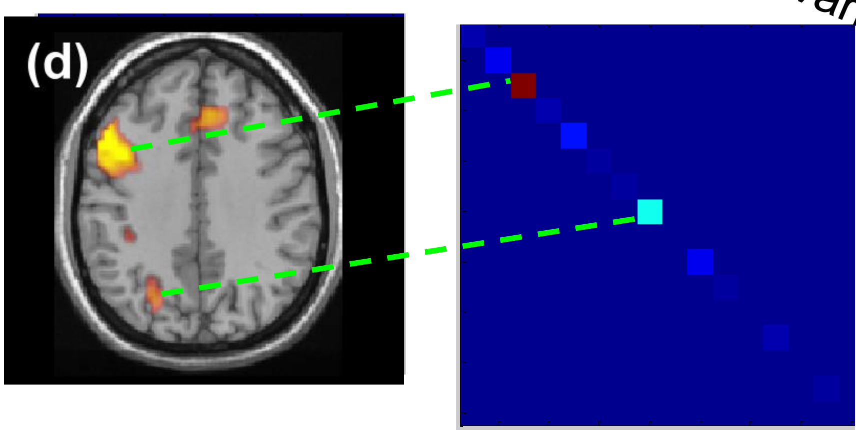


PREDICTED

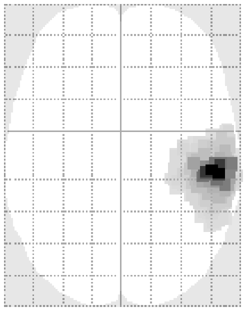


Inverse problem

Prior info (source covariance)

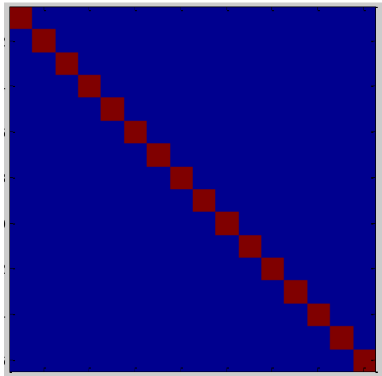


fMRI data

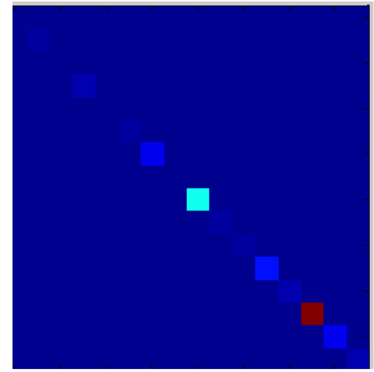


Maybe…

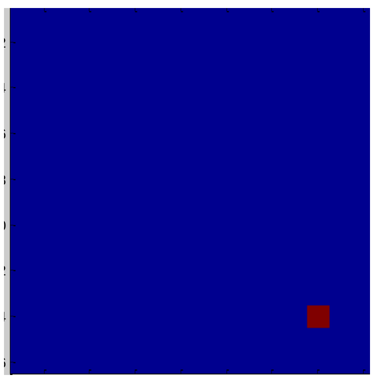
Some popular priors



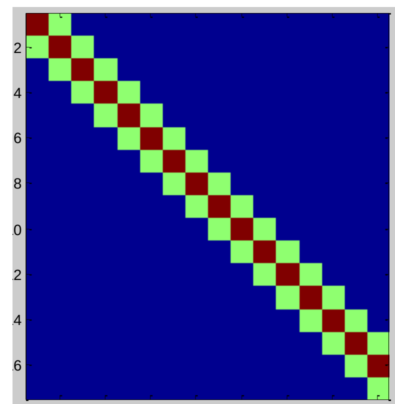
Minimum norm



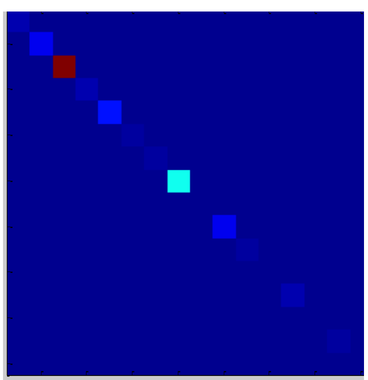
SAM,DICs
Beamformer



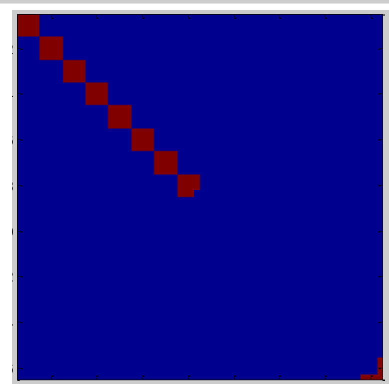
Dipole fit



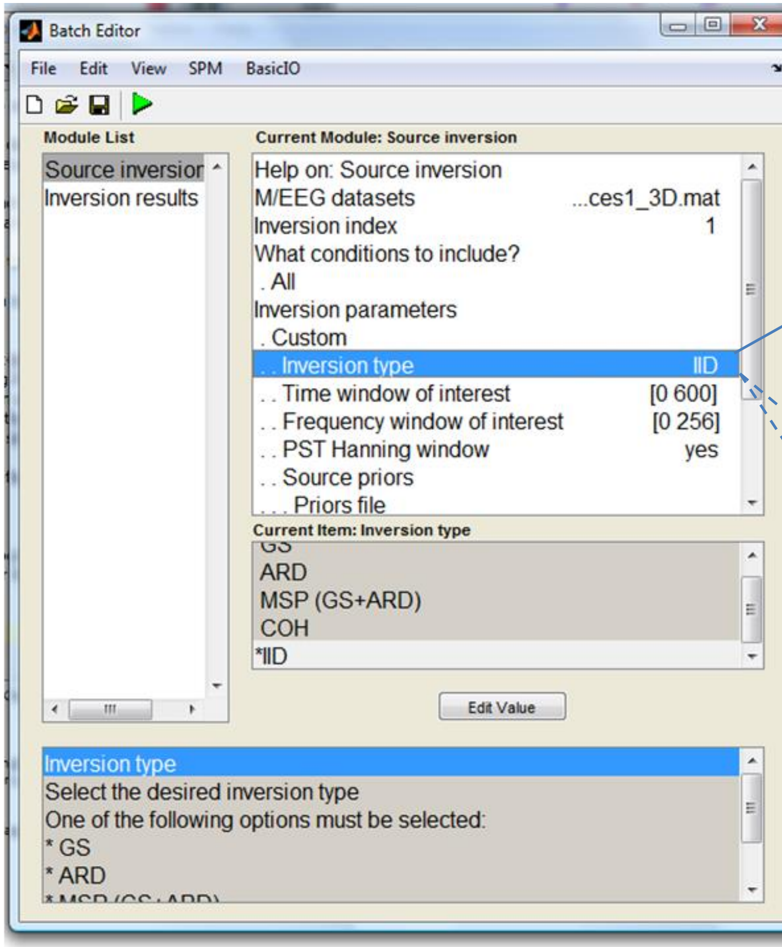
LORETA



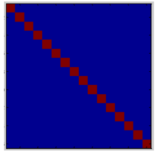
fMRI biased
dSPM



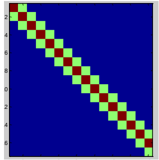
?



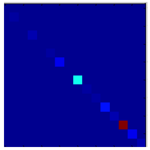
Minimum Norm (IID
- independent and identically distributed)



LORETA (COH- coherent)



Empirical Bayes Beamformer (EBB)



Multiple Sparse Priors
(MSP/ Greedy Search (GS)
Automatic relevance determination (ARD))

Summary

- MEG inverse problem requires prior information in the form of a source covariance matrix.
- Different inversion algorithms- SAM, DICS, LORETA, Minimum Norm, dSPM… just have different prior source covariance structure.
- Historically- different MEG groups have tended to use different algorithms/acronyms.

See
Mosher et al. 2003, Friston et al. 2008, Wipf and Nagarajan 2009, Lopez et al. 2013

Software

- ***SPM12: http://www.fil.ion.ucl.ac.uk/spm/software/spm12/***

- ***DAiSS-*** SPM12 toolbox for Data Analysis in Source Space (beamforming, minimum norm and related methods), developed by Vladimir Litvak:

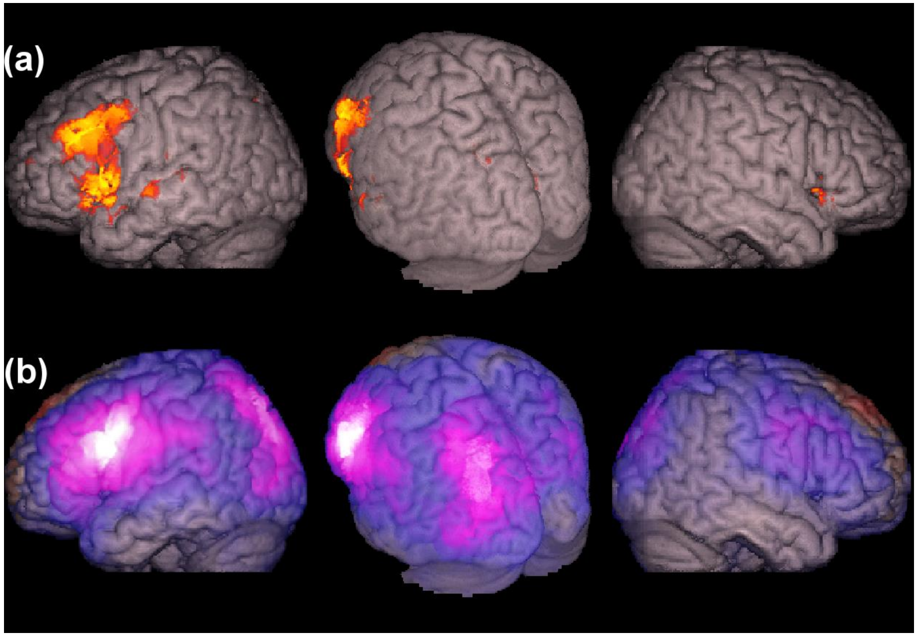
  https://github.com/spm/DAiSS

- **Fieldtrip** : http://fieldtrip.fcdonders.nl/
- **Brainstorm**:http://neuroimage.usc.edu/brainstorm/
- **MNE**: http://martinos.org/mne/stable/index.html

Which priors should I use ?

- Compare to other modalities..

fMRI



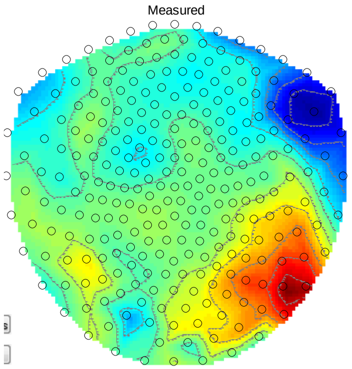
MEG
beamformer

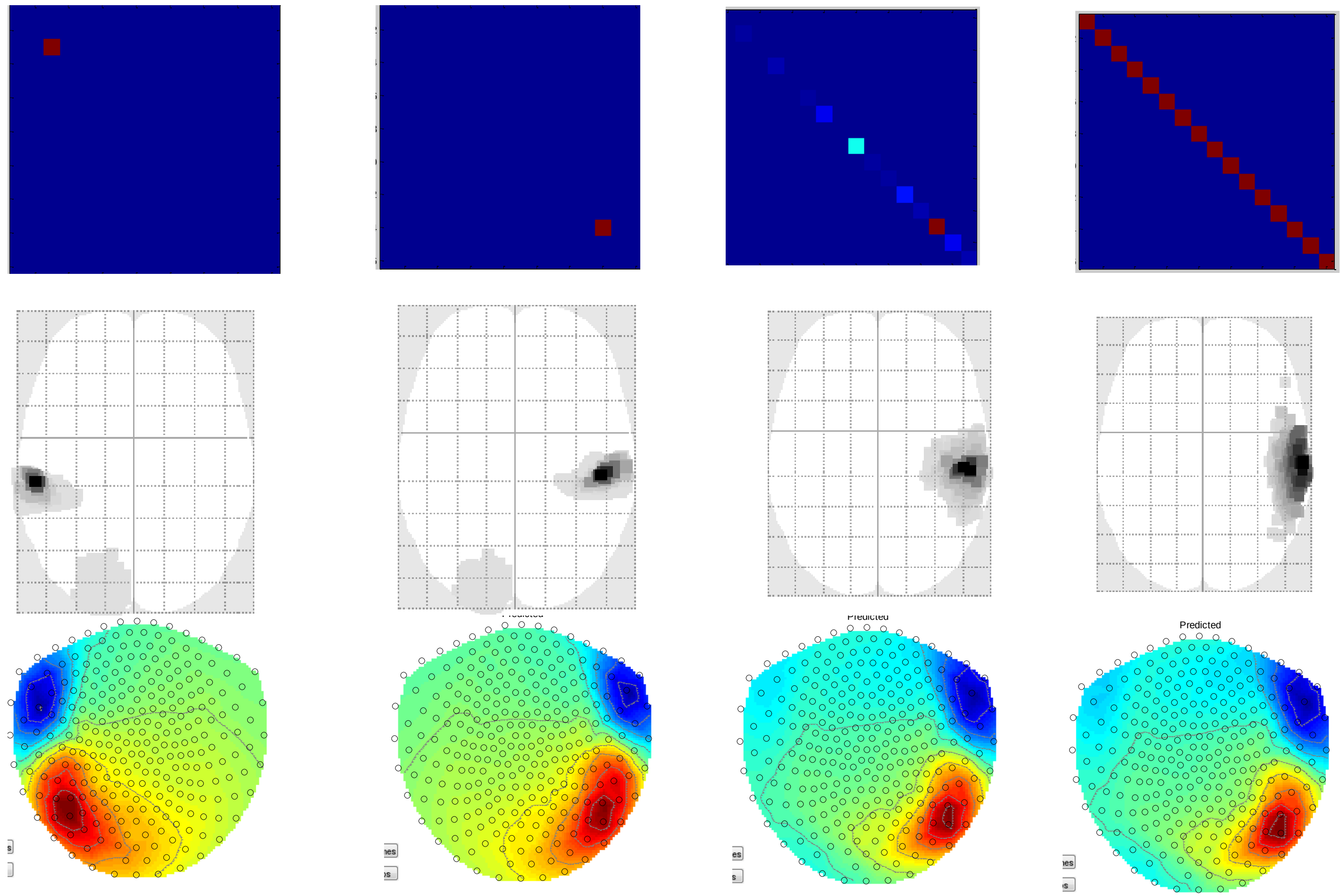
Singh et al.
2002

- Use model comparison… rest of the talk.

Y (measured field)

How do we chose between priors ?



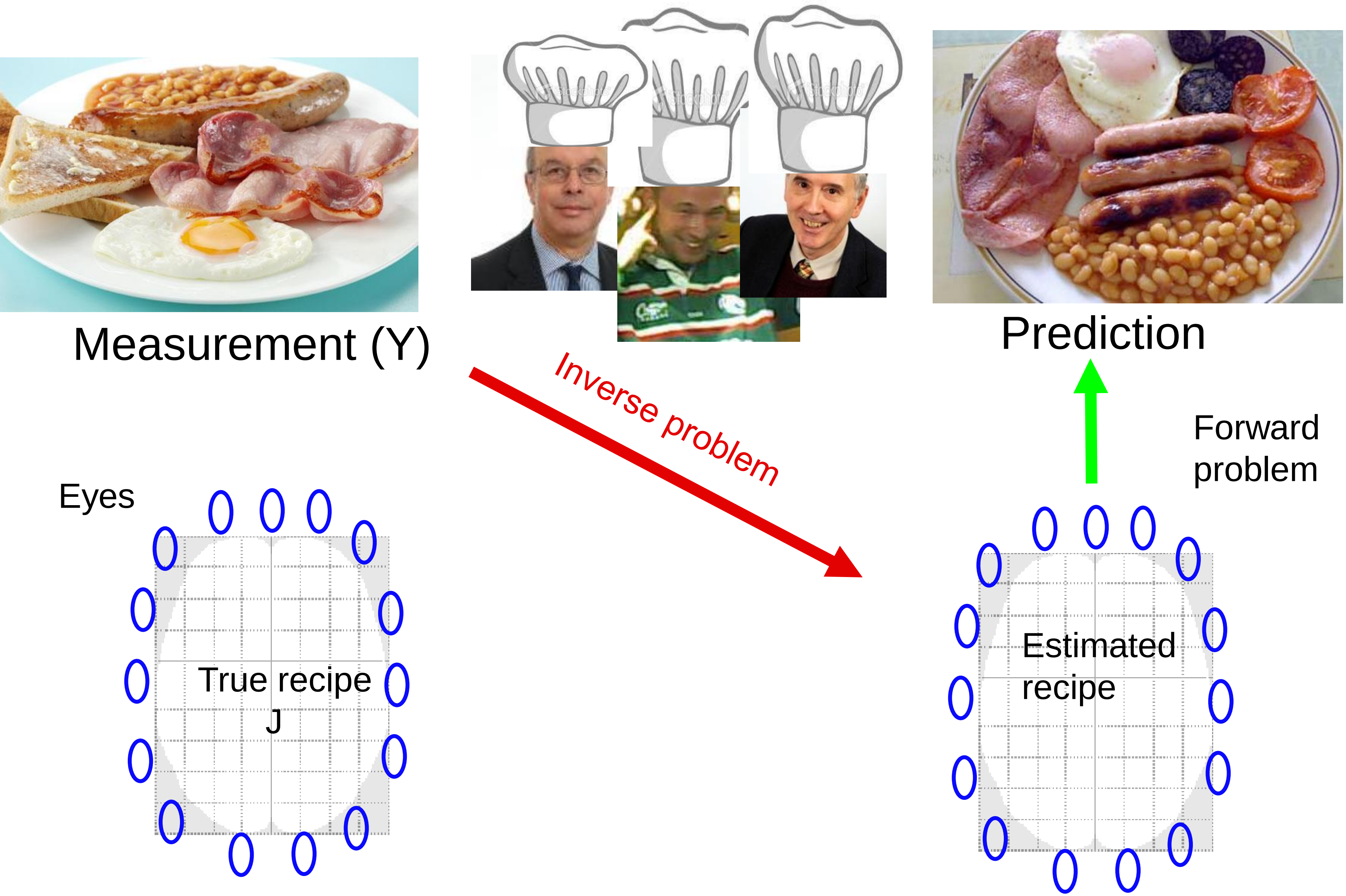


Prior

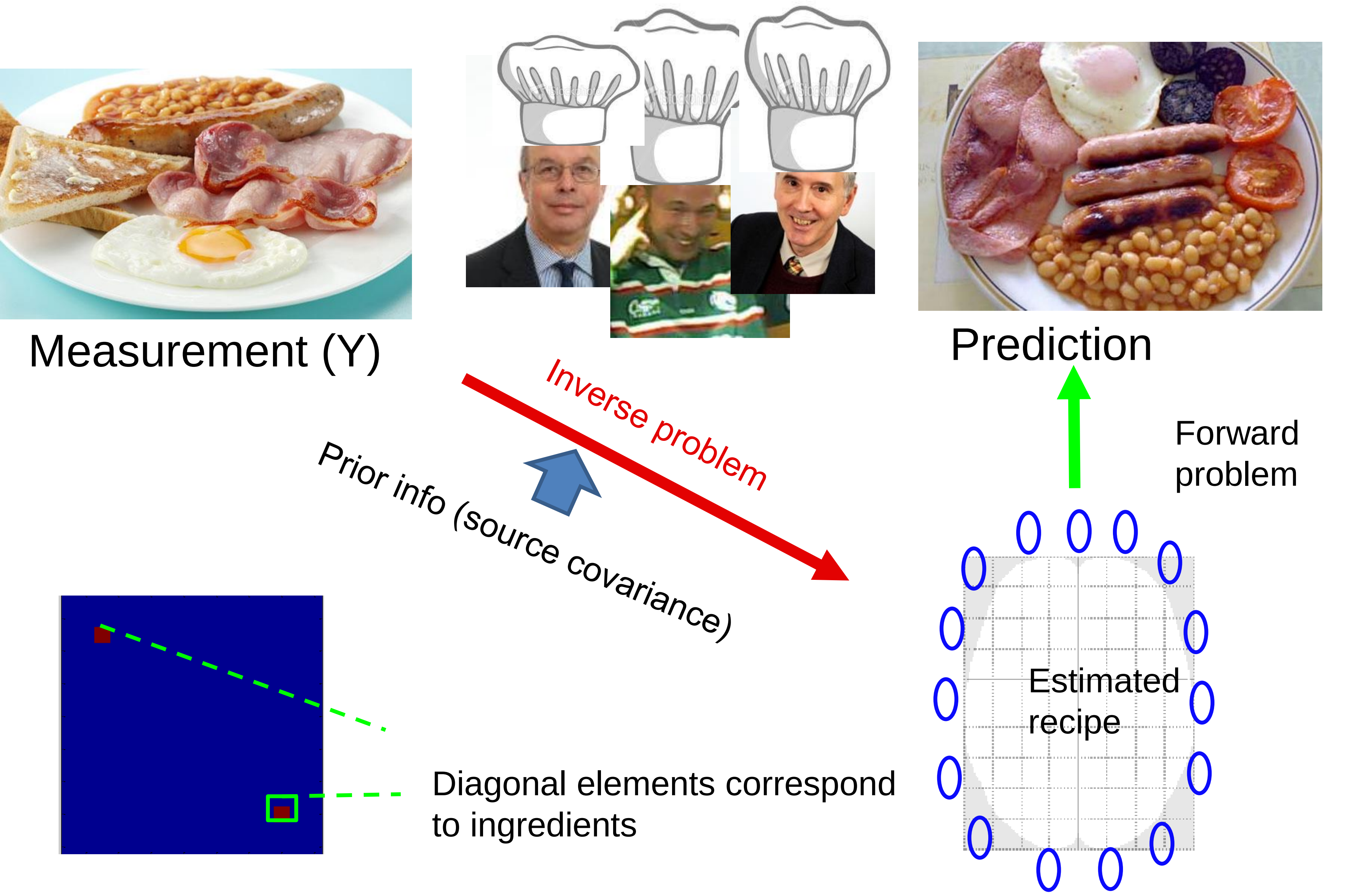
Estimated Current flow

Predicted data

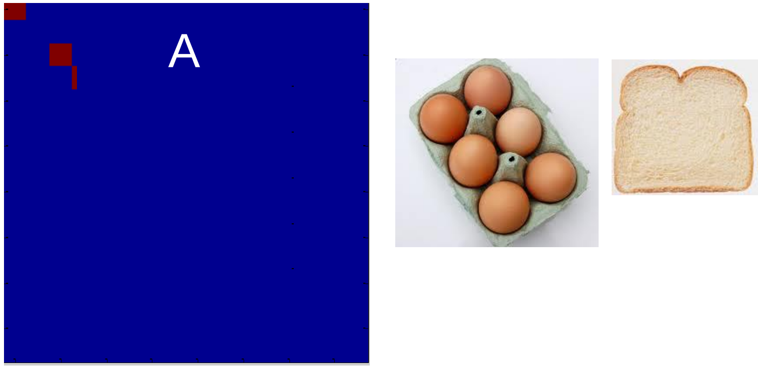
Variance explained	11 %	96%	97%	98%

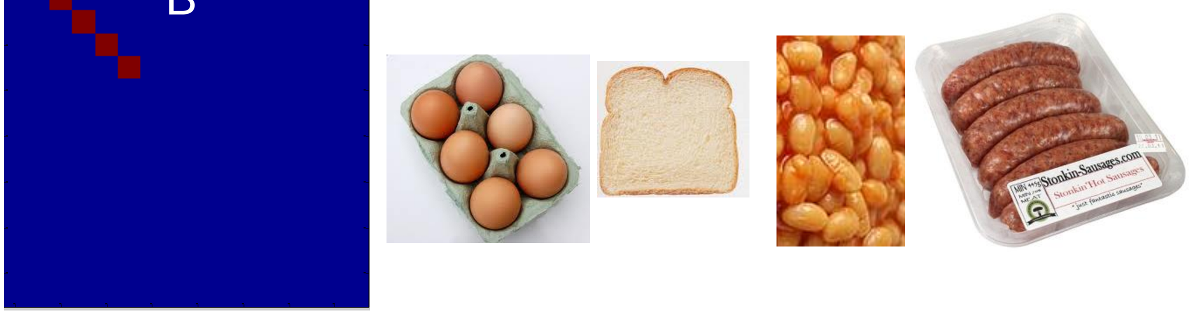

Measurement (Y)
Inverse problem
Prediction
Forward problem
Eyes
True recipe
J
Estimated recipe

Use prior info (possible ingredients)

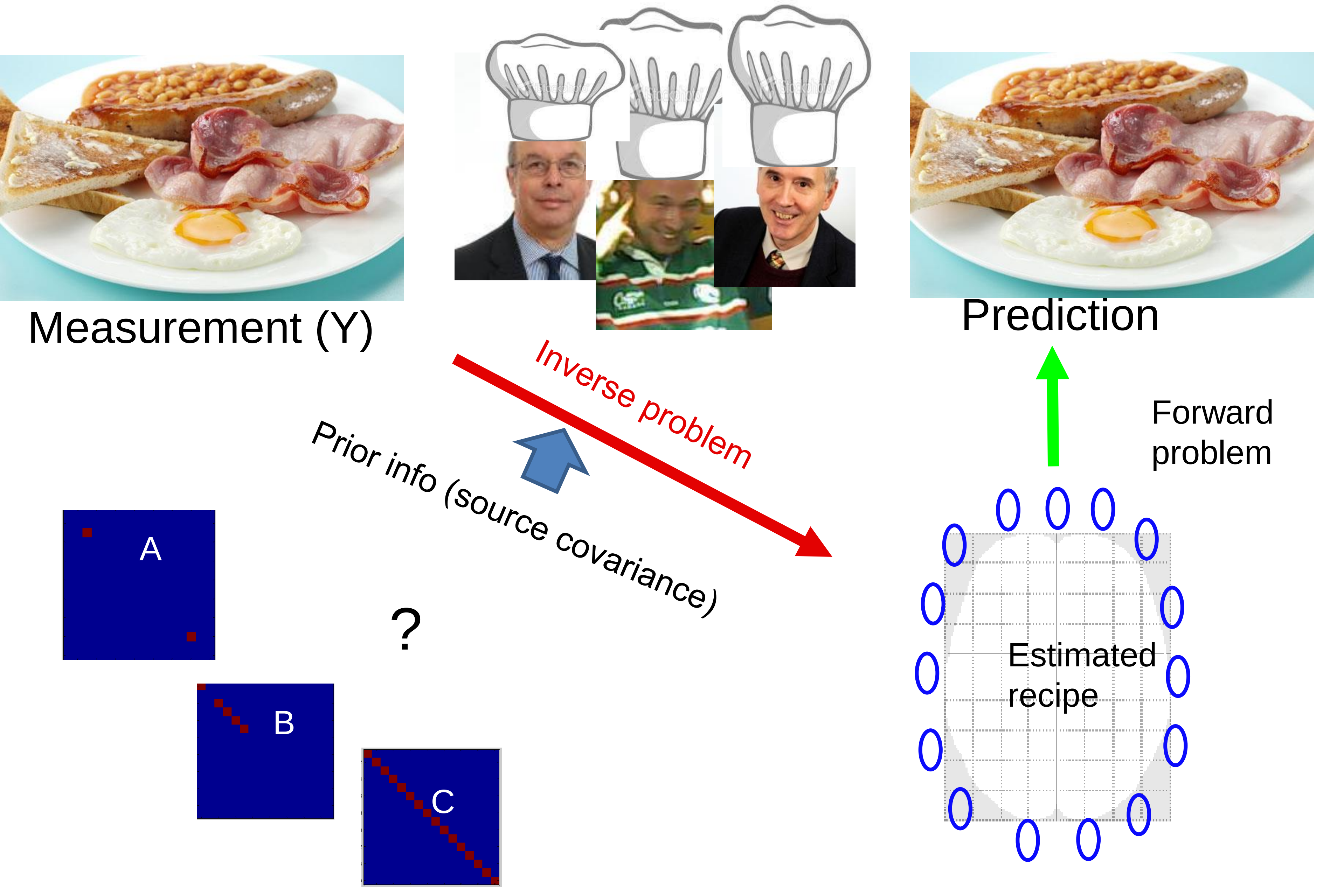


Possible priors

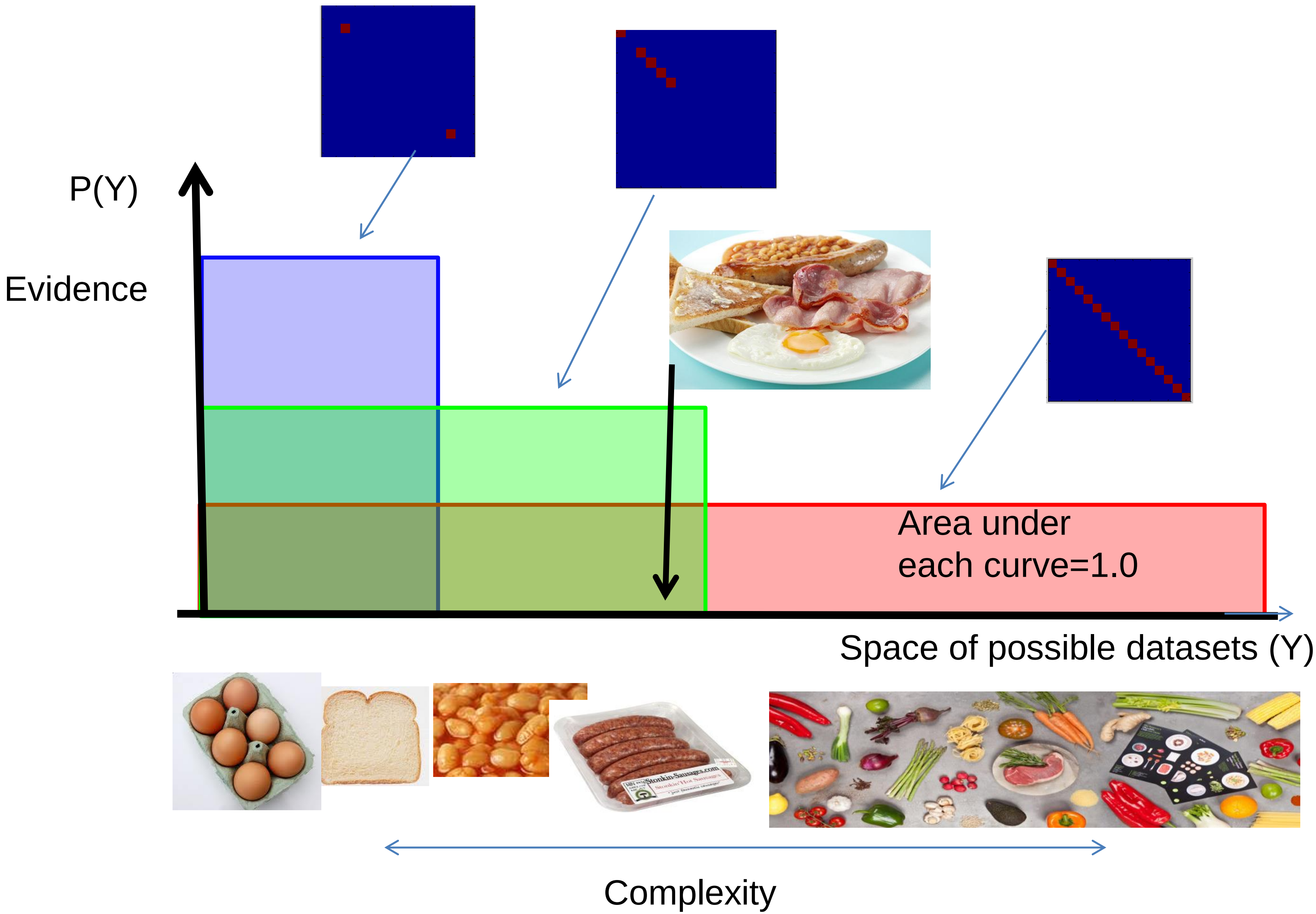




Which is most likely prior (which prior has highest evidence) ?

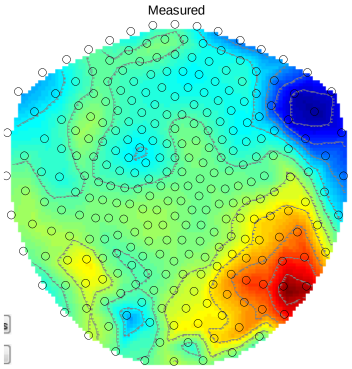


Consider 3 generative models



How do we chose between priors ?

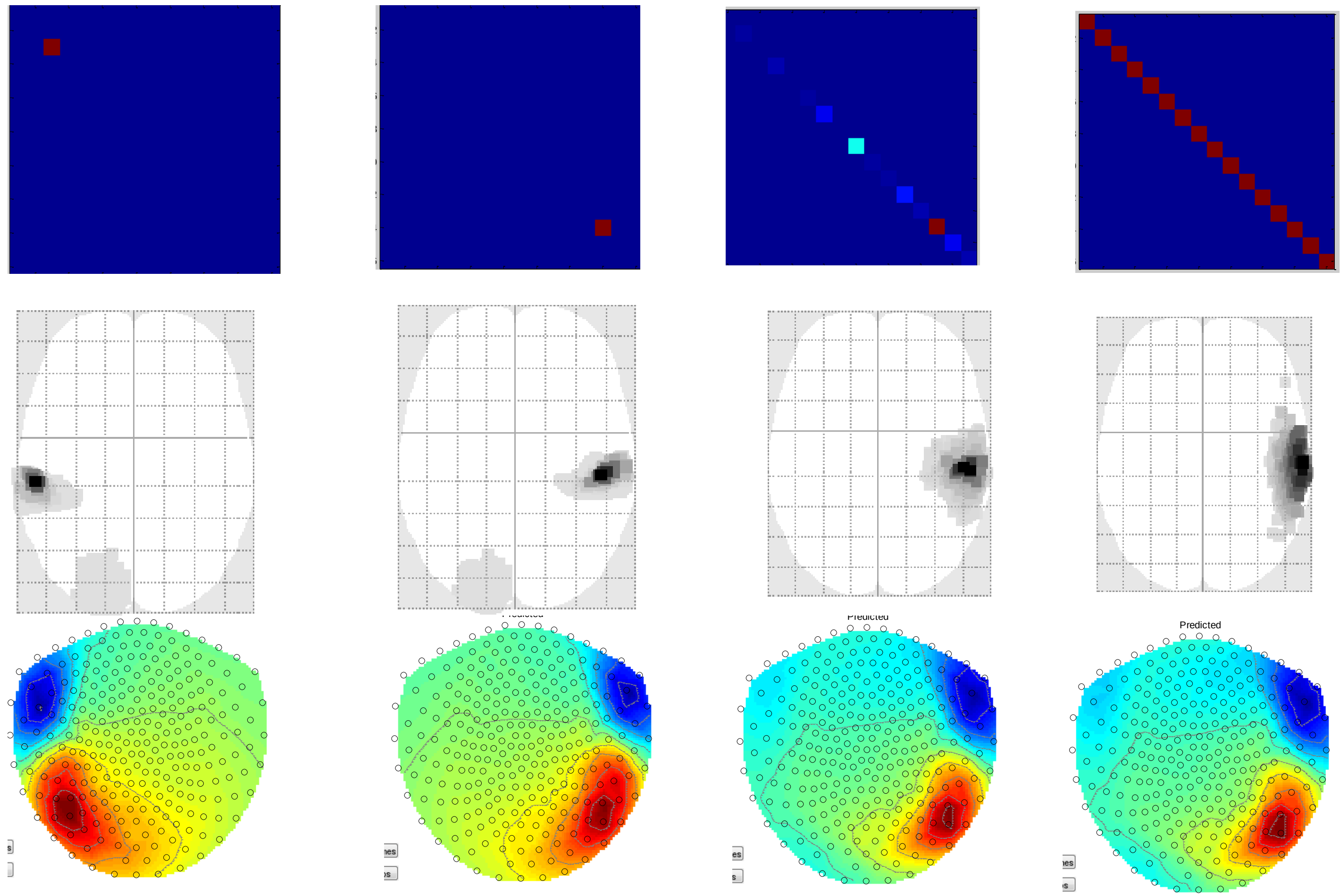
Y (measured field)



Prior

Estimated Current flow

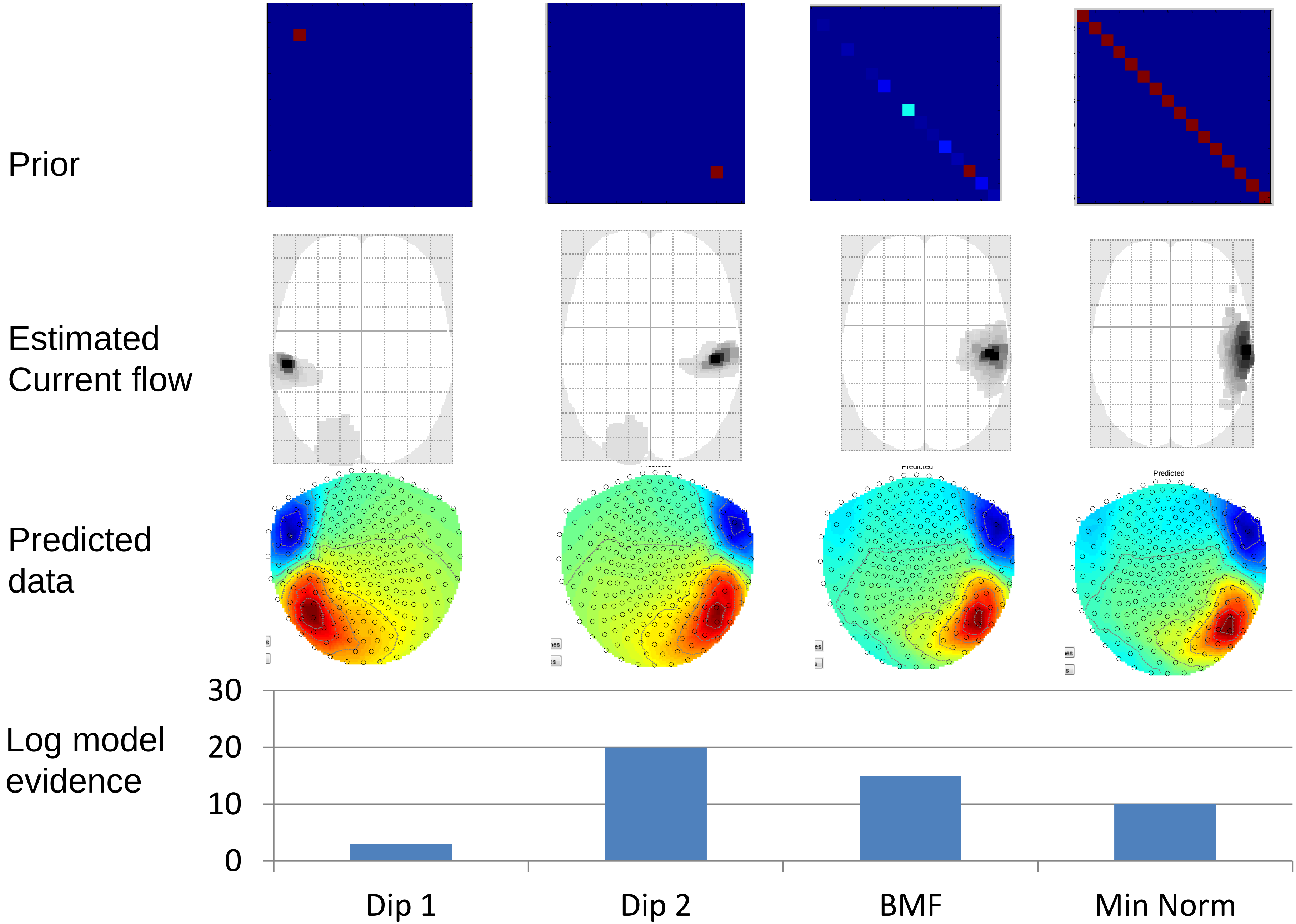
Predicted data



Variance explained

11 % | 96% | 97% | 98%

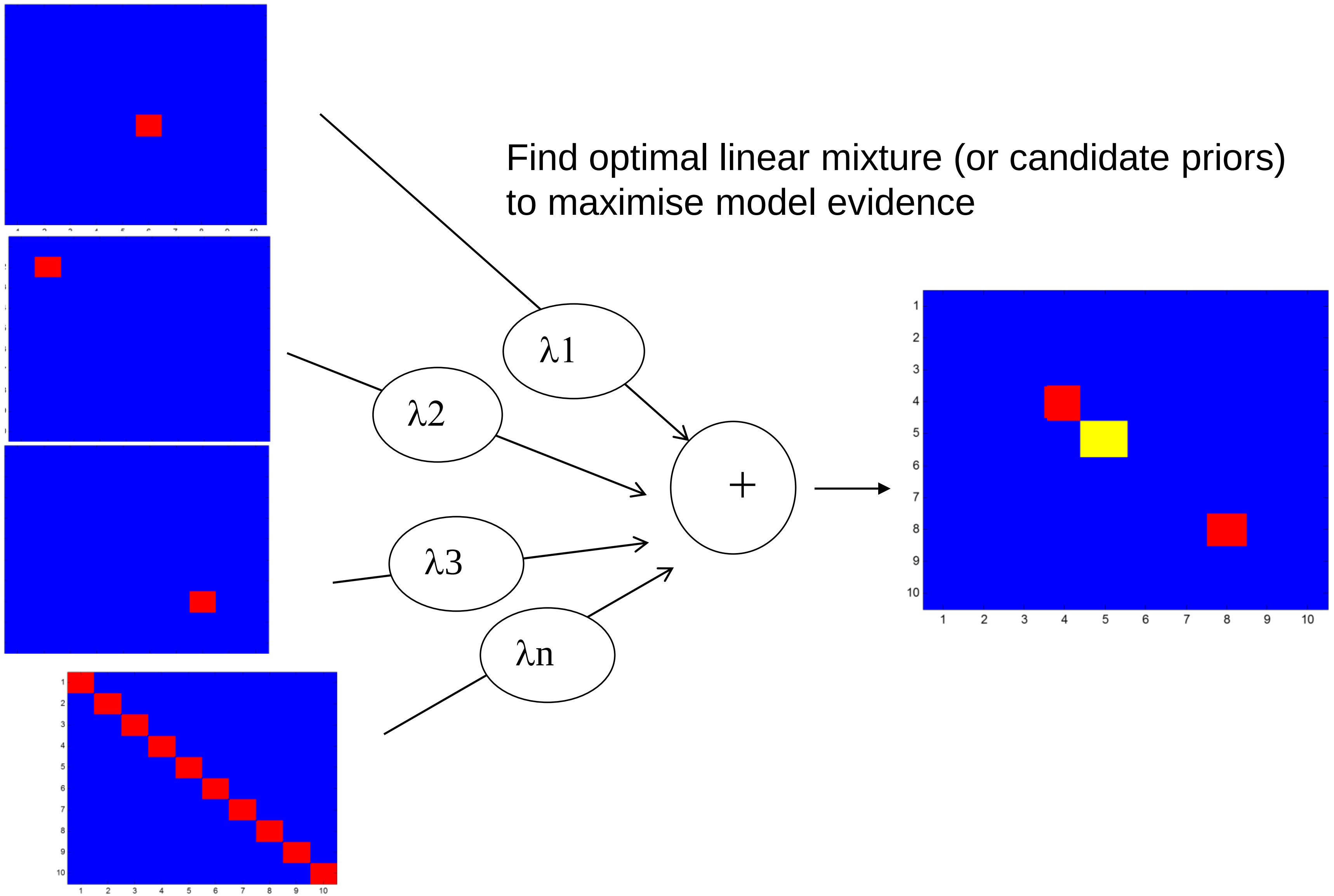
How do we chose between priors ?



Muliple Sparse Priors (MSP), Champagne

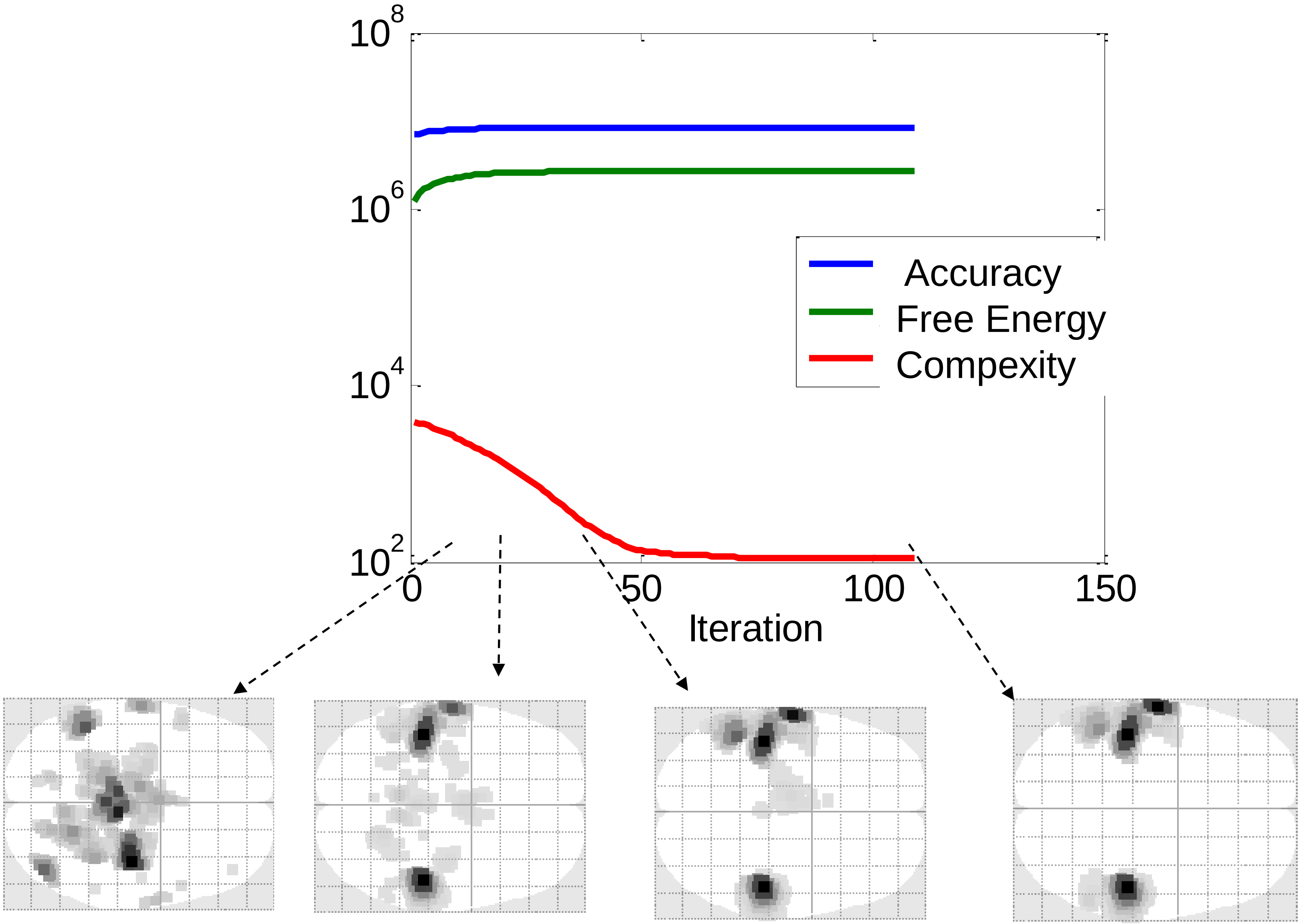
Candidate Priors

Find optimal linear mixture (or candidate priors) to maximise model evidence



Multiple Sparse priors

So now construct the priors to maximise model evidence



Key points :

- What is an ill-posed inverse problem
- Prior knowledge- links to popular algorithms.
- Validation of prior knowledge/ Model evidence

Conclusion

- M/EEG inverse problem can be solved.. If you have some prior knowledge.
- All prior knowledge encapsulated in a source covariance matrix.
- Can test between priors (or develop new priors) within a Bayesian framework.

References

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- J. Mosher, S. Baillet, R.M. Leahi
- **Equivalence of linear approaches in bioelectromagnetic inverse solutions**
- IEEE Workshop on Statistical Signal Processing (2003), pp. 294–297

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- K. Friston, L. Harrison, J. Daunizeau, S. Kiebel, C. Phillips, N. Trujillo-Barreto, R. Henson, G. Flandin, J. Mattout
- **Multiple sparse priors for the M/EEG inverse problem**
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- Wipf and Nagarajan, 2009
- D. Wipf, S. Nagarajan
- **A unified Bayesian framework for MEG/EEG source imaging**
- NeuroImage, 44 (2009), pp. 947–966

Thank you

- Karl Friston
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- Vladimir Litvak
- Guillaume Flandin
- Will Penny
- Jean Daunizeau
- Christophe Phillips
- Rik Henson
- Jason Taylor
- Luzia Troebinger
- Chris Mathys
- Saskia Helbling

And all SPM developers

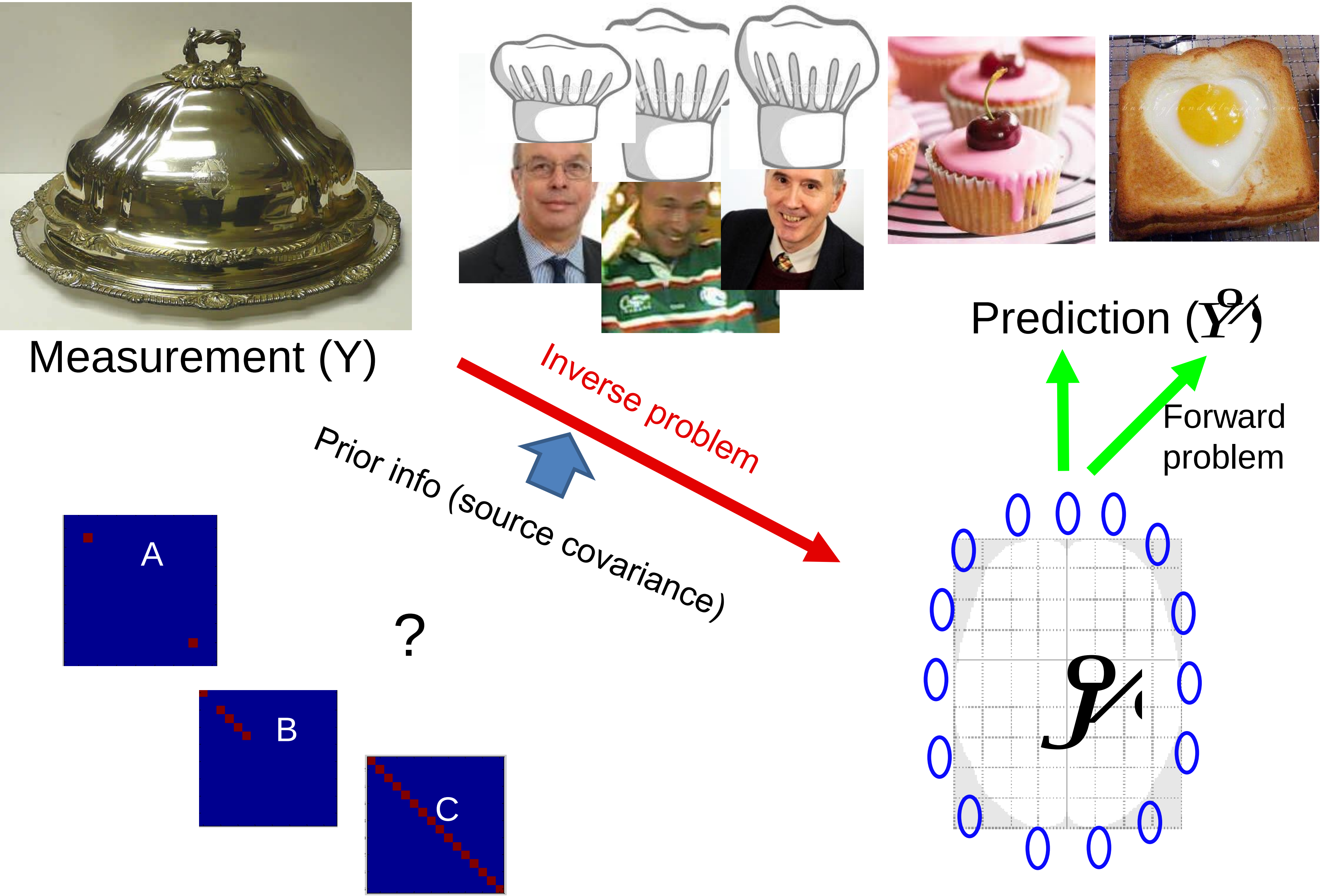
Analytical approximation to model evidence

- Free energy= accuracy- complexity

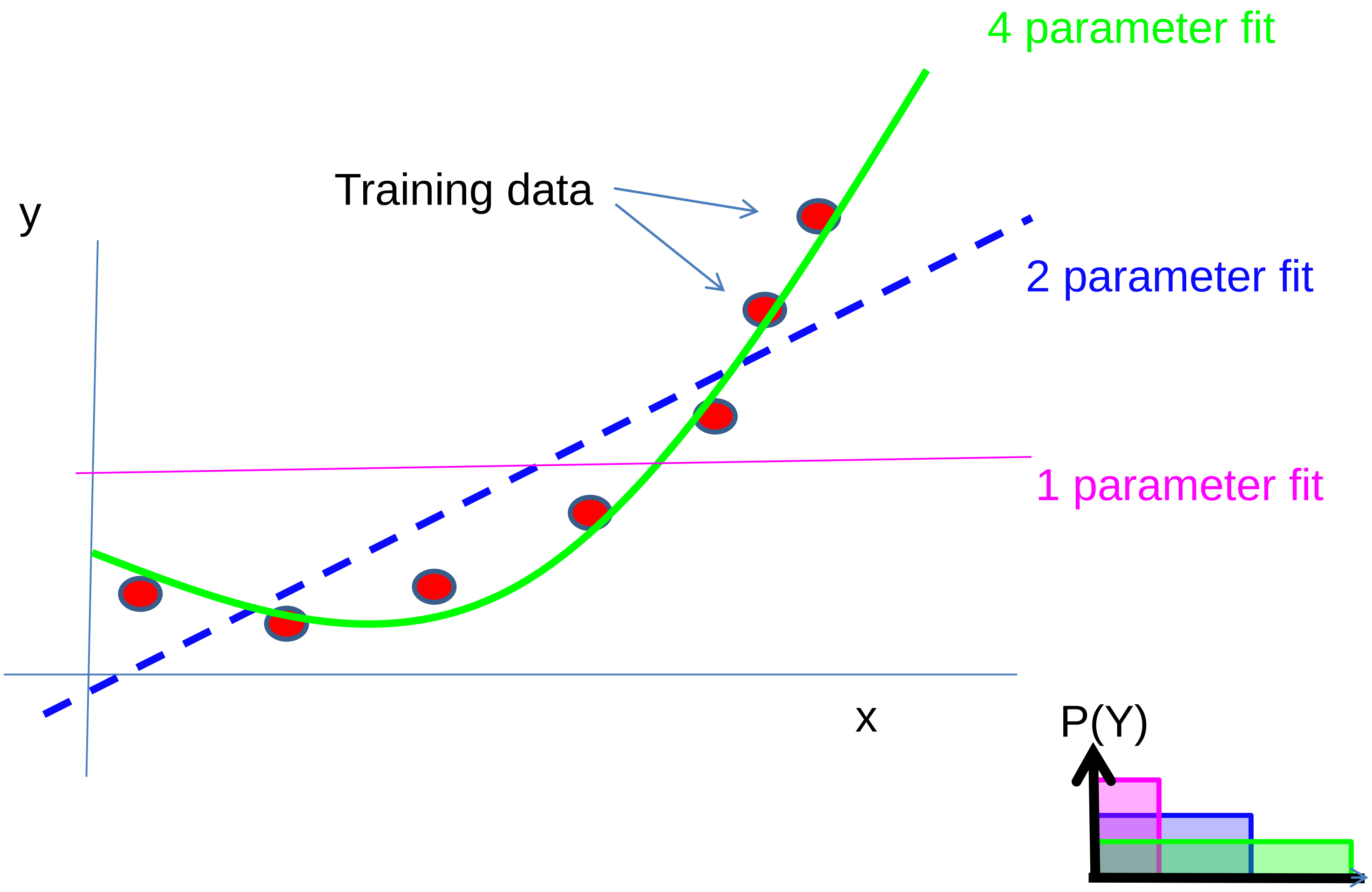
$$F = -\frac{N_n}{2}\mathrm{tr}\left(\Sigma_Y\Sigma^{-1}\right) - \frac{N_n}{2}\log|\Sigma| - \frac{N_nN_c}{2}\,log 2\pi$$
$$-\frac{1}{2}\left(\hat{\lambda}-\nu\right)^T \Pi\left(\hat{\lambda}-\nu\right) + \frac{1}{2}\log|\Sigma_\lambda \Pi|$$

$$F = -\begin{bmatrix}\text{Model}\\ \text{error}\end{bmatrix} - \begin{bmatrix}\text{Size of model}\\ \text{covariance}\end{bmatrix} - \begin{bmatrix}\text{Num of data}\\ \text{samples}\end{bmatrix}$$
$$-\begin{bmatrix}\text{Error in}\\ \text{hyperparameters}\end{bmatrix} + \begin{bmatrix}\text{Error in covariance}\\ \text{of hyperparameters}\end{bmatrix}.$$

Cross validation or prediction of unknown data

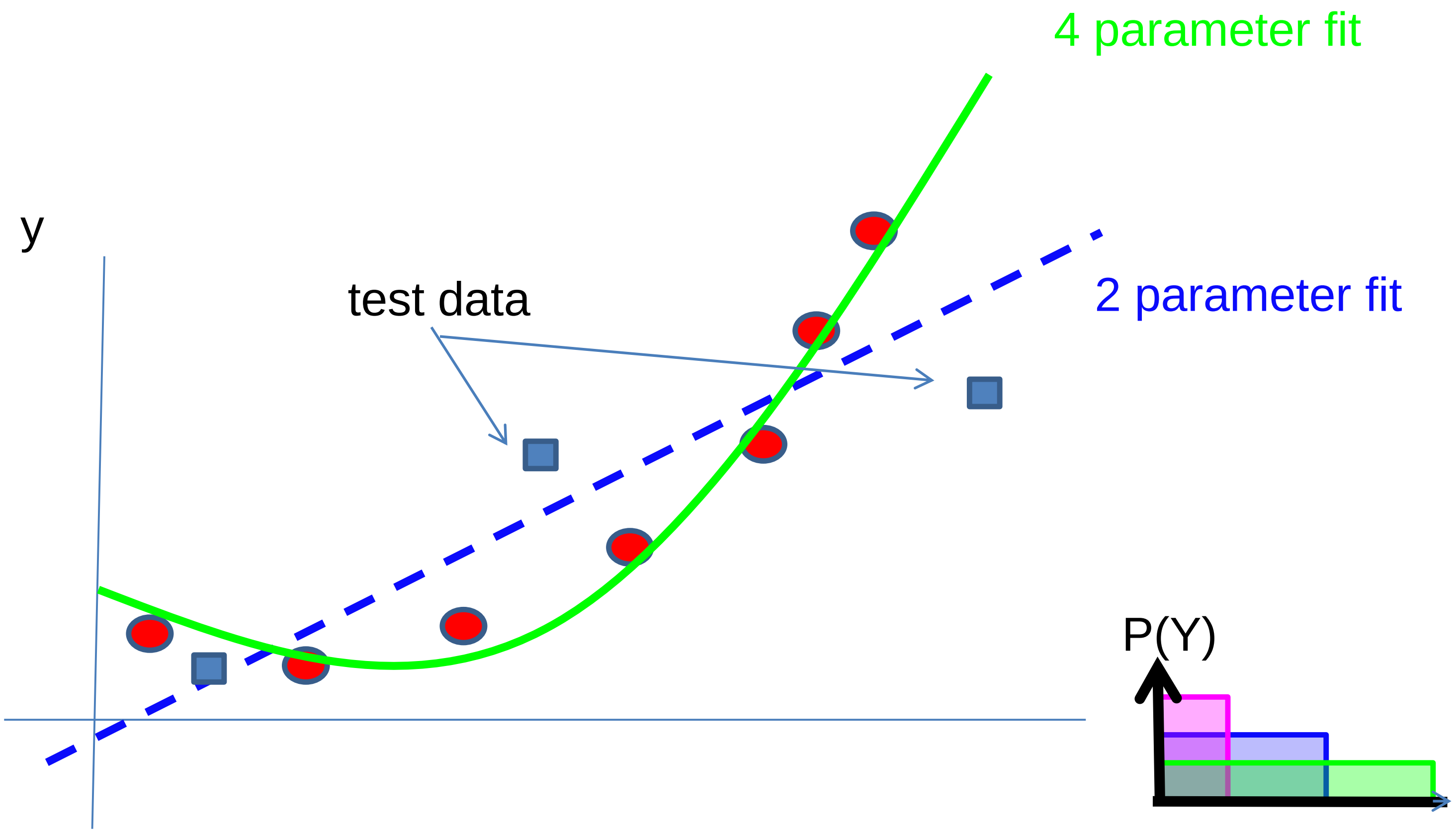


Polynomial fit example



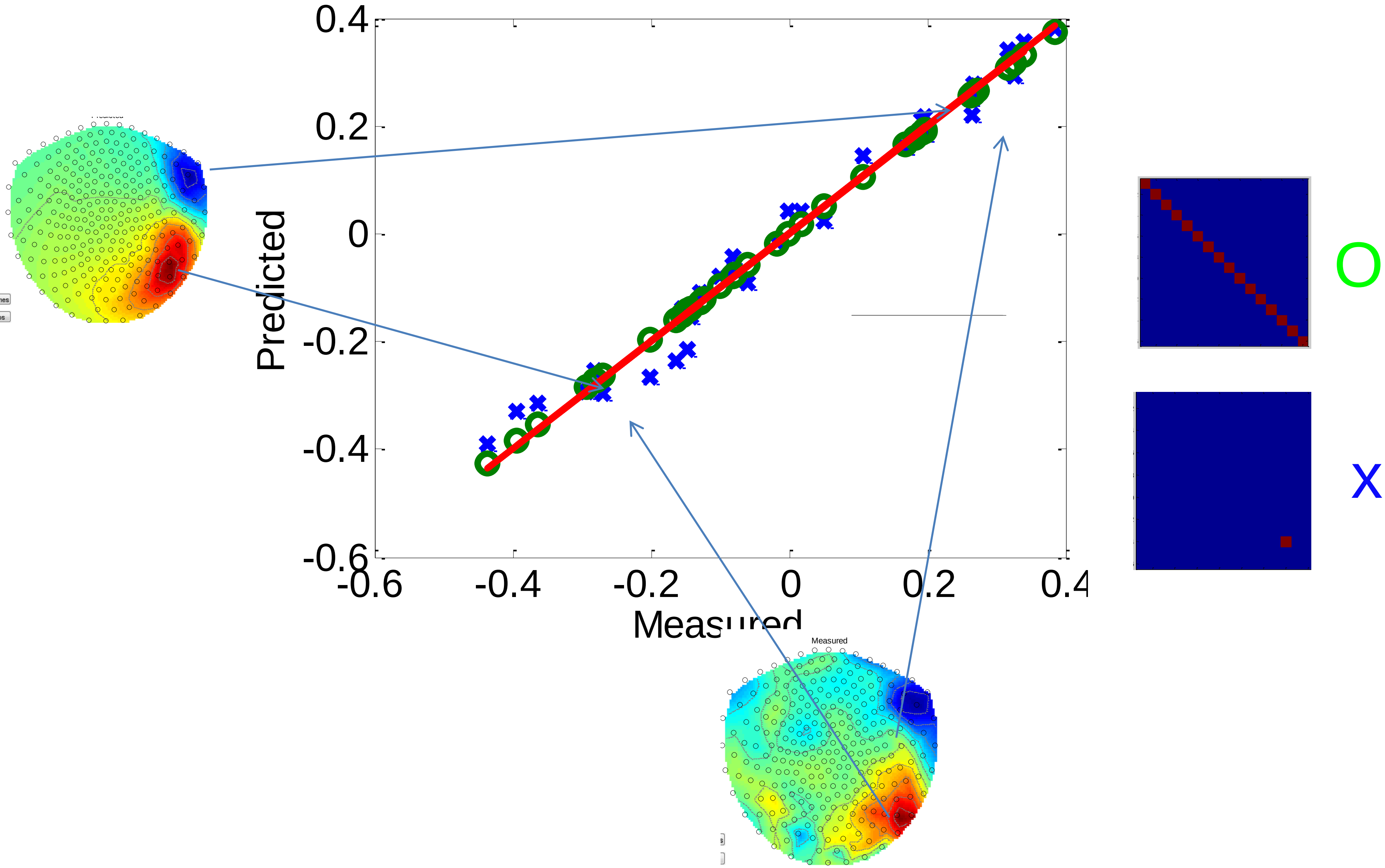
The more parameters in the model the more accurate the fit (to training data).

Polynomial fit example



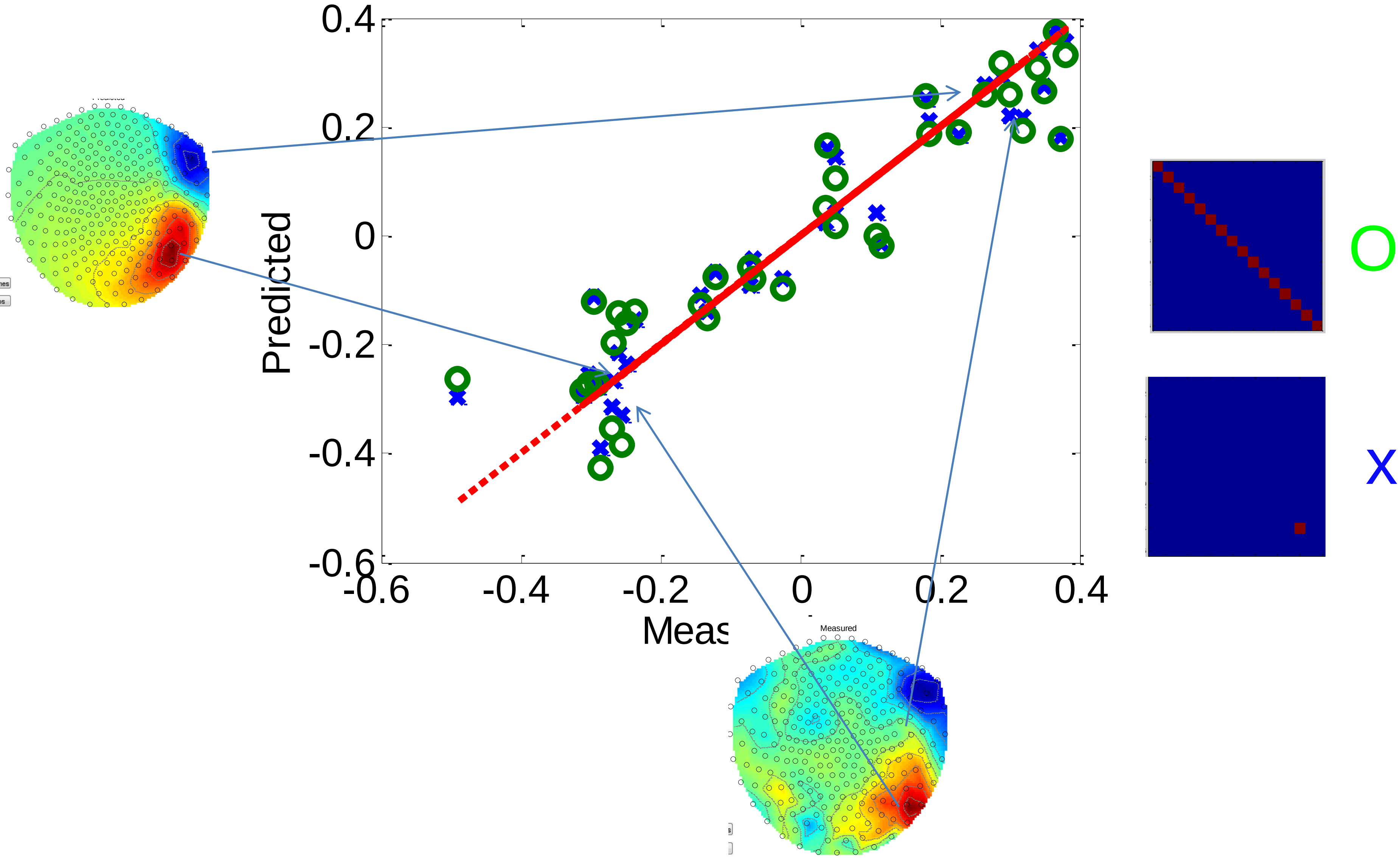
The more parameters the more accurate the fit to training data, but more complex model may not generalise to new (test) data.

Fit to training data



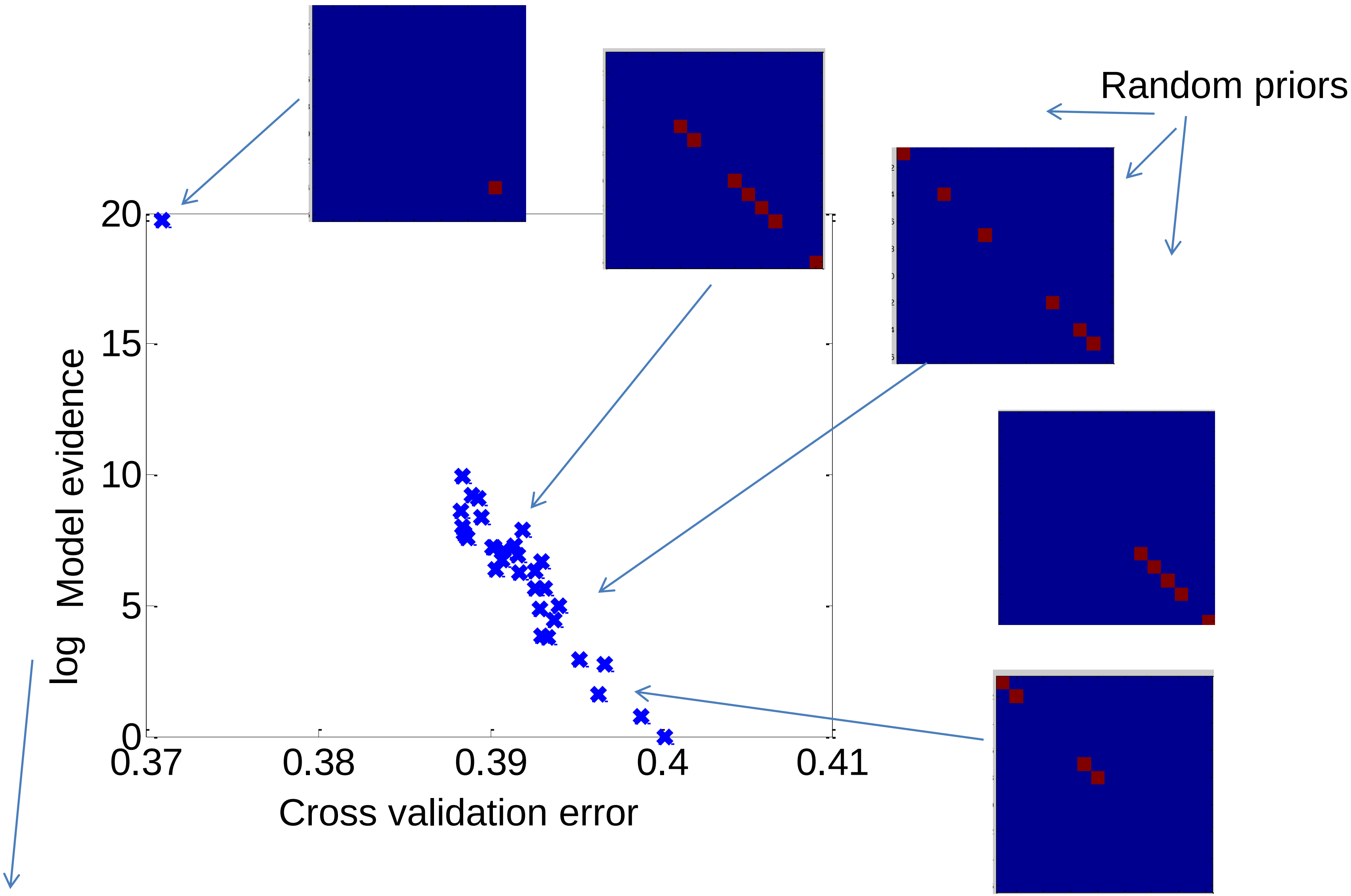
More complex model fits training data better

Fit to test data



Simpler model fits test data better

Relationship between model evidence and cross validation



Can be approximated analytically…